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A schematic model for pentaquarks based on diquarks

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Abstract

QCD instantons are known to produce deeply bound diquarks which may be used as building blocks in the formation of multiquark states, in particular, pentaquarks and dibaryons. We suggest a simple model in which the lowest scalar diquark (and possibly the tensor one) can be treated as an independent "body", with the same color and (approximately) the mass as a constituent (anti)quark. In this model a new symmetry between states with the same number of "bodies" but different number of quarks appear, in particular, the 3-"body" pentaquarks can be naturally related to decuplet baryons. We estimate both the masses and widths of such states, and then discuss the limitations of this model.

1. Introduction

The possibility of a low lying $\bar{q}q^4$ states in the P-wave (e.g., K^+n) channel fitting in the antidecuplet flavor representation of the quark model was advocated long ago by Golowich [1], along with the non-strange excited baryon $N(1710)$. A decade ago, when the $SU(3)$ version of the Skyrme model was refined, it was found to predict an *antidecuplet* $\overline{10}$ of baryons above the conventional octet and decuplet. It was not taken seriously till relatively recent works [2] which predicted among others a resonance in K^+n with a mass of 1540 MeV.

In remarkable agreement with this prediction, several recent experiments have reported an exotic baryon $\Theta^+(1540)$ with a small (and so far unmeasured) width [3]. The issue of its consistence with earlier Kd data is discussed in [4] and also [5]. The observed angular distribution suggests a likely spin-1/2 state, with so far unknown parity. Its minimal quark content is a pentaquark, i.e., $(ud)^2\bar{s}$. The antidecuplet flavor assignment was further strengthened by an observation by the NA49 Collaboration [6] of a family of exotic Ξ baryons, with a mass of 1.86 GeV and width smaller than the experimental resolution of 18 MeV.

The theoretical advantage of the Skyrme model is that it allows to reduce a complex multiquark problem into a single-body problem, with one pseudoscalar meson moving in a fixed classical background. However, the price for such reduction, based on the "large N_c ideology" maybe prohibitive given the large degeneracies implied. The $1/N_c$ description implies a small width, that is difficult to assess quantitatively given the subtleties related to these corrections [7].

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More traditional "shell model ideology" (e.g., the MIT bag model or nonrelativistic constituent quark models) tends to put as many quarks as possible into the lowest shell, and thus predict *negative* parity, $P = -1$ for the lowest state.[1]

The shell model works well for nuclei; in this case the pairing effects are small and treated perturbatively using the shell model states. However, we think the order should be reversed for hadrons, and pairing into diquarks be treated first. One argument for that is that the many flavor-symmetric exotic states possible in a shell model have never been seen. Even the most symmetric "magic" configuration, the dibaryon $H = u^2d^2s^2$, an analogue of the alpha particle, appears to be not deeply bound, as it was never found in multiple dedicated searches.

As we will argue in this Letter, the picture most consistent with the current new findings are those developed in a "small N_c ideology", in which the key element are the *instanton-induced*[2] diquarks [10,11]. Due to the Pauli principle at the level of instanton zero modes, two quarks of the same flavor cannot interact with the same instanton. The propagation of 5 quarks through the QCD vacuum generates many interactions involving 't Hooft interaction, some second-order ones are depicted in Fig. 1. The latter illustrates the strong preference for multiquark states to be in the lowest possible flavor representation, avoiding many other possible exotic states, both in the meson and baryon sectors. As we will argue, even these newly discovered states, although truly exotic, still are in a way analogous to the decuplet baryons. Their small decay widths is a consequence of a *different internal structure*, with small overlap with all the decay channels.

For a review on the instanton vacuum models one can consult [11]. The main approximations are: (i) a reduction of the gauge configurations to the subset of instantons and antiinstantons; (ii) a focus on only the fermionic states that are a superposition of their zero modes. When the baryonic (3-quark) correlators have been first calculated in it [12] a decade ago (and soon confirmed by lattice measurements [13]) a marked difference between the nucleon (octet) and Δ (decuplet) correlators has been noted. Roughly speaking, a nucleon was found to be made of a quark and a very deeply bound *scalar–isoscalar diquark*, absent in the decuplet. As it was found to have a surprisingly small mass comparable to the constituent quark mass (to be denoted below as Σ), it significantly simplifies the model to be discussed below.

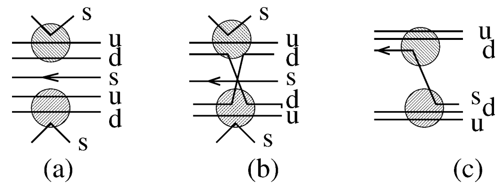

Fig. 1. Some second-order instanton-induced interactions of 5 quarks propagating in time through the (Euclidean) QCD vacuum. The shaded circles indicate instantons and antiinstantons. The quarks are avoiding quarks of the same flavor and 3-body force is repulsive, so (a) is the diagram generating two independent diquarks. The instantons have to pick up pairs from the vacuum condensate $\langle\bar{s}s\rangle$ to get it attractive. The diagram (b) with a light quark exchange generates a repulsive core, while the diagram (c) leads to diquark attraction.

The general theoretical reason for the lightness of the scalar–isoscalar diquark state (see, e.g., [14]) follows from the special Pauli–Gursey symmetry of 2-color QCD. In this theory (the "small N_c limit" of QCD) the scalar diquarks are actually *massless Goldstone bosons*. For general N_c, the instanton (gluon-exchange) in qq is $1/(N_c - 1)$ down relative to $\bar{q}q$. So the real world with $N_c = 3$ is half-way between $N_c = 2$ with a relative weight of 1, and $N_c = \infty$ with relative weight 0. Loosely speaking, the scalar–isoscalar diquarks are *half Goldstone bosons* with a binding energy of about half of the mass, or about one constituent quark mass.

Diquarks in the context of Nambu–Jona-Lasinio models were investigated, e.g., in [15], which also emphasized the occurrence of a light scalar–isoscalar bound state. Diquark correlations have been a driving idea behind a view of dense baryonic matter as a very strong color superconductor [14,16]. If one views the nucleon as a quark plus a Cooper pair, such a view of dense matter is indeed very natural. In hadronic

[1] The lattice studies by Csikor et al. [8] and Sasaki [9], indeed, claim a signal for $P = -1$ pentaquarks with a comparable mass. More and better data are, however, needed to reach firm conclusions on the matter.

[2] Although scalar diquarks are also attracted by single-gluon exchange forces, the latters do not lead to the structure we discuss as they are flavor-blind.

spectroscopy the nonet of scalar mesons below 1 GeV is believed to be made of diquark–antidiquark states.

In such a context it is even more natural to see the pentaquarks as an antiquark plus *two* Cooper pairs. Jaffe and Wilczek (JW) [17] (see also Nussinov [4]) have already suggested to view the $\Theta^+(1540)$ as an object made of 2 diquarks $(ud)(ud)\bar{s}$, where (ud) is a *scalar*–isoscalar diquark in relative P-wave. This model leads to an $\mathbf{8}_f \oplus \overline{\mathbf{10}}_f$ flavor representation for the pentaquark states. They also argued that the long-known Roper 1440, may also be a $(ud)^2\bar{d}$ pentaquark state belonging to an octet. In a more recent paper [18] they have added further considerations following from the Na49 cascade data: the most important one is that they seem to provide experimental indications on the existence of the pentaquark octet, together with $\overline{\mathbf{10}}$.

In this Letter we develop these ideas a bit further, suggesting a schematic model which has enough symmetries to allow estimates of the pentaquark masses by relating them to those of decuplet baryons. Our input are the values for the "diquark masses", calculated in the random instanton liquid model (RILM).[3]

2. $\bar{\mathbf{3}}_c$ diquarks

All diquarks to be discussed below are *antitriplets* in color (both instanton and gluon interactions are repulsive in the sextet) with generic spin–flavor assignments as follows:

$$(q\boldsymbol{\Gamma}q)^a = \epsilon_{abc} q_b^{\mathrm{T}} \boldsymbol{C}\boldsymbol{\Gamma} q_c, \tag{1}$$

where $\boldsymbol{C}$ is the charge conjugation matrix, and $\boldsymbol{\Gamma}$ include the pertinent Dirac and flavor matrices. Diquarks with all possible Dirac matrices Γ in $q^{\mathrm{T}}C\Gamma q$ have been studied in RILM [12]. The pseudoscalar channel with $\Gamma = 1$ was found to be very strongly repulsive, the vector and axial vector channels are weakly repulsive, with a mass of the order of 950 MeV, above twice the constituent quark mass of the model, $2\Sigma = 840$ MeV. The only two channels with attraction and significant binding are: (i) the *scalar* with $m_S \approx \Sigma$ and $\Gamma = \gamma_5$; (ii) the *tensor* with $m_T \approx 570$ MeV and $\Gamma = \sigma_{\mu\nu}$ (denoted below by a subscript T).[4] The scalar is odd under spin exchange while the tensor is even under spin exchange. Fermi statistics forces their flavor to be different. The scalar is flavor-antisymmetric $\bar{3}$ while the tensor is flavor-symmetric 6.

In the model to be discussed below, we will discuss all possible pentaquark multiplets which can be made using these ingredients. For scalar diquarks we will introduce the following shorthand notation in $SU(3)_f$

$$\underline{S} = (u^{\mathrm{T}}C\gamma_5 d), \qquad \underline{U} = (s^{\mathrm{T}}C\gamma_5 d), \\ \underline{D} = (u^{\mathrm{T}}C\gamma_5 s). \tag{2}$$

3. Model

We treat diquarks on equal footing with constituent quarks. Because of their similar mass and quantum numbers, certain approximate symmetries appear between states with the same numbers of "bodies". This simple idea is depicted pictorially in Fig. 2. The $\bar{q}q$ mesons (a) are a well-known example of the 2-body objects, as well as the quark–diquark states (b) (the octet baryons $q\underline{q}$). The diquark–antidiquark states (c) are in this model the 2-body objects. In *zeroth* order, the usual non-strange mesons (like ρ, ω), the octet baryons (like the nucleon), and the 4-quark mesons (like $a_0(980)$)[5] are degenerate, with a mass $M \approx 2\Sigma = 840$ MeV. To *first* order, which includes color-related interactions, the one-gluon-exchange Coulomb and confinement, the degeneracy should still hold, as the color charges and the masses of quarks and diquarks are the same. Only in *second* order, when the spin–spin and other residual forces are included, they split. There is no spin–spin interaction for the nucleon (the scalar diquark has no spin), while for the ρ it is either repulsive (if it is due to one gluon exchange) or zero (if it is due to the instanton-induced forces [20]). Note that this new symmetry between N, ρ and $a_0(980)$ is actually rather accurate, better than the old $SU(6)$ symmetry, stating (in zeroth order) that $M_N \approx M_\Delta$.

[3] Those exist as physical hadrons only in $N_c = 2$ QCD. However, since the instanton liquid model does not confine, there are diquark states for any N_c.

[4] The longitudinal vector diquark channel with $\Gamma = \gamma_\mu\gamma_5$ mixes with the scalar $\Gamma = \gamma_5$ in the P-wave. This point is relevant to the lattice studies discussed in [9].

[5] For recent study of these states in the instanton model see [19].

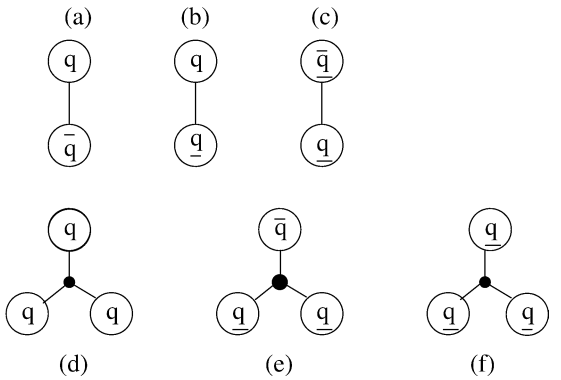

Fig. 2. Schematic structure of (a) ordinary mesons, (b) quark–diquark or octet baryons, (c) diquark–antidiquark states or tetraquarks, (d) decuplet baryons, (e) pentaquarks and (f) dibaryons.

Pentaquarks in such a model are treated as 3-body objects, with two correlated diquarks plus an antiquark, and thus there are simple relations between masses of various "3-body objects" depicted in Fig. 2(d)–(f) with the "3-body" (octet/decuplet) baryons.

From the color point of view, all 3-body states involve the same ϵ_{abc} wave function, just like the ordinary color singlet baryons. From the flavor point of view the situation is different. For pentaquarks made of two scalar diquarks the flavor representations are $\bar{3} \otimes \bar{3} \otimes \bar{3} = 1 \oplus 8 \oplus 8 \oplus \overline{10}$. Using the notations we introduced above, and changing from bar to underline where needed, one can readily see how the pentaquarks observed fit onto an antidecuplet, $\Theta^+(1540) = (ud)(ud)\bar{s} = \underline{SS}\bar{s}$ is an analogue of anti-Ω, and is thus the top of the antidecuplet (the conjugate of the decuplet). New exotic $\Xi(1860)$ are $\underline{UU}\bar{u}$ and $\underline{DD}\bar{d}$, providing the two remaining corners of the triangle. They are the analogue of anti-Δ. The remaining 7 members can mix with one octet, as discussed by Jaffe and Wilczek, making together 18 states in flavor representations $(8 \oplus \overline{10})$. For ordinary 3 quarks there is the overall Fermi statistics which ties together flavor- and spin-space symmetry and works against the remaining $1 \oplus 8$. There is no such argument for pentaquarks. So how are the additional flavor states $1 \oplus 8$ excluded for pentaquarks?

For diquark–diquark–antiquark all there is left is Bose statistics for identical scalars, demanding total symmetry over their interchange, while the color wave function is antisymmetric. So the only solution [4,17] is to make the spatial wave function antisymmetric by putting one of the diquark into the P-wave state. It means that such pentaquarks should be degenerate with the excited P-wave decuplet baryons:

$$M_\Theta = 2\Sigma + \Sigma_s + \delta M_{L=1} + V_{\text{residual}}, \tag{3}$$

where the first 2 terms are masses of the diquarks and strange quark, plus an extra contribution for the P-wave, plus whatever *residual* interaction there might be.

It is straightforward to assess $\delta M_{L=1}$ by analogy with the P-wave baryon excitations. Indeed, the diquark mass is about the constituent quark mass, and the confining potential is also the same. For example, following the well-known paper by Isgur and Karl [21] one can simply use an oscillator potential, in which the separation of the center of mass motion from the internal motion is relatively simple. Introducing three standard Jacobi coordinates, one finds that the difference between P-wave and S-wave state is $\delta M_{L=1} = \hbar\omega_\lambda \approx 480$ MeV. Very similar values were obtained using more modern constituent quark models, e.g., a semi-relativistic model with a linear potential by the Graz group [22],[6] so we consider our assessment justified.

Ignoring for the time being all residual interactions, one may estimate the pentaquark mass to be that of a decuplet baryon with a single s plus the P-wave penalty, i.e.,

$$\begin{aligned} m_\Theta &\approx m^*_\Sigma(3/2) + \delta M_{L=1} \\ &\approx 1400 + 480 = 1880 \text{ MeV}, \end{aligned} \tag{4}$$

which is well above the observed mass of 1540 MeV.

However, using one scalar and one *tensor* diquark one can do without the P-wave penalty, and the schematic mass estimate now reads

$$\begin{aligned} m_\Theta &\approx m^*_\Sigma(3/2) + \delta M_T \\ &\approx 1400 + 150 = 1550 \text{ MeV}, \end{aligned} \tag{5}$$

which is much closer to the experimental value.

The newly observed $\Xi(1860)$ pentaquarks contains diquarks with a strange quark, that is us, ds. Their

[6] The difference with Isgur and Karl is in the nature of the spin–spin forces which are not important for scalar (spinless) diquarks.

masses have not been yet directly calculated, but a general experience with spin-dependent forces [20] suggests a reduction of binding by about a factor 0.6 as compared to the ud case. This suggests a total loss of binding of about 200–240 MeV, which together with a strange quark mass itself (two s-quarks instead of a single $\bar{s}$) readily explains the 320 MeV mass difference between $\Xi(1860)$ and $\Theta^+(1540)$ pentaquarks.

Since the tensor diquark has the opposite parity, both possibilities correspond to the same global parity $P = +1$. Also common to both schemes is the fact that the total spin of 4 quarks is 1, so adding the spin of the $\bar{s}$ can lead not only to $s = 1/2^+$ but also to $s = 3/2^+$ states (which are not yet observed).

So, we conclude that if we only look at the *masses*, it appears that it is better to substitute one diquark by its tensor variant, rather than enforce the P-wave. However, such an alternative scheme provides a different set of flavor representations as we now show. Indeed, $\bar{3} \otimes 6 \otimes \bar{3} = 1 \oplus 8 \oplus 8 \oplus 10 \oplus 27$. The largest representation 27 has particles with quantum numbers of Θ^+ and $\Xi(1860)$, and even more exotic triplets such as Ω-like $sssq\bar{q}$ states. The cascades have isospin $3/2$, as observed. However, Θ^+ is a part of an isotriplet, with Θ^{++} and Θ^0 partners. The former can decay into pK^+, a quite visible mode, in which no resonance close to 1540 was seen. Since the widths are unknown at this point, it is perhaps premature to conclude that they do not exist. However, if the occurrence of the decay mode $\Theta^{++} \to pK^+$ is definitively ruled out, the observed multiplet of exotics cannot be the 27.

The Roper resonance belongs to octet with the quark content $\underline{SS}\bar{d}$. In the JW model with the P-wave, its mass would be estimated as

$$\begin{aligned} m_{\text{Roper}} &= 3\Sigma + \delta M_{L=1} \\ &\approx m_\Delta + \delta M_{L=1} = 1260 + 480 \\ &= 1740 \text{ MeV}, \end{aligned} \tag{6}$$

while in a variant with the tensor diquark it is only

$$\begin{aligned} m_{\text{Roper}} &= 3\Sigma + \delta M_T \\ &\approx m_\Delta + \delta M_T \\ &\approx 1260 + 150 = 1410 \text{ MeV}, \end{aligned} \tag{7}$$

which once again gets us closer to the experimental value. However, this corresponds to 27 flavor representation, where its isospin is $3/2$. In the lower 8 flavor representation with isospin $1/2$, it will include $\bar{s}s$ and be too heavy again.

4. Widths and Goldberger–Treiman relations

Small widths are not the consequence of the centrifugal barrier, as the P-wave is not really producing sufficiently small factors. As we already mentioned, a general argument for small pentaquark widths is small overlap between the internal and external (KN) wave functions. In this section we make this relation more explicit.

The decay widths including Goldstone bosons are determined by general properties of their chiral interaction, and expressions can be somewhat simplified. The strong decay of the pentaquark $P(\frac{1}{2}^+) \to \pi N(\frac{1}{2}^+)$ is conditioned by a generalized Goldberger–Treiman relation. The one-pion reduced axial vector current has a transition matrix

$$\begin{aligned} &\langle P(p_2)|\boldsymbol{j}^a_{A\mu}(0)|N(p_1)\rangle \\ &\quad = \bar{P}(p_2)\big(\gamma_5\gamma_\mu \boldsymbol{G}(t) + (p_2 - p_1)_\mu \bar{\boldsymbol{H}}(t)\big)\frac{\tau^a}{2}N(p_1), \end{aligned} \tag{8}$$

with $\boldsymbol{j}^a_{A\mu}$ partially conserved [23],

$$\partial^\mu \boldsymbol{j}^a_{A\mu}(x) = f_\pi\big(\Box + m_\pi^2\big)\pi^a(x). \tag{9}$$

The first form factor in (8) is one-pion reduced with $\boldsymbol{G}(0) = \boldsymbol{g}_{PN}$ the “axial overlap” charge. If its value be close to the axial charge of the nucleon, it would mean that pentaquark is nothing but a PN system. However, as we will see, the data demand it to be significantly smaller.

Inserting (9) into (8) gives

$$\begin{aligned} &\langle P(p_2)|\pi^a(0)|N(p_1)\rangle \\ &\quad = \frac{1}{f_\pi}\frac{1}{m_\pi^2 - t} \\ &\qquad \times \bar{P}(p_2)\big((m_P + m_N)\boldsymbol{G}(t) + t\bar{\boldsymbol{H}}(t)\big)\frac{\tau^a}{2}N(p_1). \end{aligned} \tag{10}$$

By definition, the pseudoscalar π–PN coupling is

$$\begin{aligned} &\langle P(p_2)|\pi^a(0)|N(p_1)\rangle \\ &\quad = \boldsymbol{g}_{\pi PN}(t)\frac{1}{m_\pi^2 - t}\bar{P}(p_2)\gamma_5\tau^a N(p_1), \end{aligned} \tag{11}$$

which corresponds to

$$\boldsymbol{g}_{\pi PN}\pi^a(\bar{P}\tau^a N + \text{h.c.}).$$

A comparison of (11) to (10) gives at the pion pole $t \approx m_\pi^2$

$$f_\pi \boldsymbol{g}_{\pi PN}(m_\pi^2) + \sigma_{\pi PN}(m_\pi^2) = \frac{m_P + m_N}{2}\boldsymbol{g}_{PN}(m_\pi^2), \tag{12}$$

which is the general form of the Goldberger–Treiman relation for the transition amplitude $P \to N\pi$. The overlap sigma-term is proportional to m_π^2/Λ, which is typically 40 MeV in the pion–nucleon system.

The generic form of the decay width $P \to \pi N$ is given by

$$\Gamma_{P\to\pi N} = \frac{\boldsymbol{g}_{\pi PN}^2}{4\pi}\frac{q_P}{M_P}\left(\sqrt{q_P^2 + m_N^2} - m_N\right), \tag{13}$$

where q_P is the meson momentum in the rest frame of the P state,

$$M_P = \sqrt{q_P^2 + m_N^2} + \sqrt{q_P^2 + m_\pi^2}. \tag{14}$$

The recently observed $\Xi(1860)$ can be used in conjunction with (12) to bound the transition axial-overlap $\boldsymbol{g}_{PN}$ and the coupling $\boldsymbol{g}_{\pi PN}$ in the antidecuplet, thereby allowing a prediction for the width of the $\Theta(1540)$ through (13). Indeed, if we assign a conservative decay width of about 20 MeV to $\Xi^{--} \to \Xi^-\pi^-$ in light of the bound of 18 MeV reported by [6], then (12) suggests $\boldsymbol{g}_{\Xi\Xi} \approx 0.25$ and $\boldsymbol{g}_{\pi\Xi\Xi} \approx 3.75$ for $\sigma_{\pi\Xi} \approx 40$ MeV. Similar arguments yield $\boldsymbol{g}_{\Xi\Sigma} \approx 0.25$ and $\boldsymbol{g}_{K\Xi\Sigma} \approx 2.97$, thus an estimated partial width of 6.60 MeV for $\Xi^{--} \to \Sigma^- K^-$. Similarly, we would expect $\boldsymbol{g}_{\Theta N} \approx 0.25$ and $\boldsymbol{g}_{K\Theta N} \approx 2.35$, and we therefore predict a very *narrow* width of 2.60 MeV for the decay $\Theta^+ \to K^+ n$.

The narrowness of the partial widths in the antidecuplet follows from a generically small transition axial-charge smaller than 1/4, resulting into a π–PN decay constant smaller than 3 in the antidecuplet. The smallness of the axial-charge follows from the small overlap between the three and five quark states.

5. Summary and discussion

We started by emphasizing that instanton-induced 't Hooft interaction imply diquark substructure of multiquark hadrons and dense hadronic matter, with marked preference to the lowest flavor representations possible. We then summarized the finding of Ref. [12]: in the instanton liquid model whereby there are two kinds of deeply bound diquarks, the scalar and the (less bound) tensor.

We have then developed a schematic additive model, whereby diquarks appear as building blocks, on equal footing with constituent quarks. In such a model pentaquarks are treated as 3-body states, so that their classification in color and flavor becomes analogous to that of the baryons. If one uses two scalar diquarks, as suggested by Jaffe and Wilczek, the P-wave is inevitable which seems to produce states heavier than the ones reported, even in a simple additive model with very light diquarks. If one uses one scalar and one tensor diquarks, the masses look more reasonable. However, then the ensuing flavor representations are large, and although recently discovered quartet of $\Xi(1860)$ fits very well into this model, the Θ^+ has (so far) unobserved partners.

We have related the widths with the "axial overlap" charge, and have argued that current data restrict it to be significantly smaller than the nucleon axial charge, by at least a factor of 3. This means that the Skyrme-model interpretation of pentaquarks, as a Goldstone boson moving on top of the baryon is inadequate.

If one goes a step further, to 6-quark states, for example, by combining the proton and the neutron, one gets 3 ud diquarks. Again the asymmetric color wave function asks for another asymmetry: to do so one can put all 3 diquarks into the P-wave state, with the spatial wave function $\epsilon_{ijk}\partial_i\underline{S}\partial_j\underline{S}\partial_k\underline{S}$ suggested in the second paper of [14]. This will cost $3(\Sigma + \delta M_{L=1}) = 2700$ MeV, well in agreement with the magnitude of the repulsive nucleon–nucleon core. However, if one considers the quantum numbers of the famous H dibaryon, one can also make those out of diquarks such as $\underline{SDU}$. The resulting wave function is overall flavor-antisymmetric with all diquarks in S-states. Thus there is no need for P-wave or tensor diquarks for the H dibaryon. Our schematic model would then lead to a very light H, in contradiction to both experimental limits and lattice results.

This last observation calls for the lesson with which we would like to conclude our Letter: all schematic models (including our own) assume additivity of the constituents. However, as we emphasized in Fig. 1, due to the Pauli exclusion principle one instanton can only make one deeply bound diquark at a time. Thus, there must be a *diquark–diquark repulsive core*. One particular 3-body instanton repulsion effect was already discussed for the H in [24]. Multi-body instanton induced interactions were also observed in heavy-light systems [25]. A generic way to address these effects would be some dynamical studies, directly antisymmetrizing 5 or 6 quarks themselves, as well as with those in the QCD vacuum (unquenching). The evaluation of the pertinent correlators on the lattice is badly needed: studies of inter-diquark interactions in the instanton liquid model will be reported elsewhere [26]. Only with the resulting core potential included, the diquark-based description of multiquark states and of dense quark matter may become truly quantitative.

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